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INITIAL ORBIT DETERMINATION FROM LEAST SQUARES REDUCTION OF GEOCENTRIC POSITION VECTORS

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-649
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496L SYSTEM PROGRAM OFFICE
ELECTRONIC SYSTEMS DIVISION
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UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Mass.

Prepared under Contract No. AF 19(628)-562 by Aeronutronic, a Division of Ford Motor Company, Newport Beach, California

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FOREWORD

Contractor's Report Publication No. U-2442

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ABSTRACT

A computer program for initial orbit determination based on a theory employing a linear least squares fit to the geocentric position vectors has been developed. Observations can be spread over a number of days since this prodecure does not use the times of observations; however, they are used in the differential correction procedure. This program coded for the Philco 2000, is designed for operational use at the Space Detection and Tracking Center in Colorado Springs, Colorado.

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SECTION 1

INTRODUCTION

The computer program discussed in this report computes and differentially corrects the preliminary orbital elements of a satellite when given an aggregate of observation position fixes (maximum of 150). The Initial Orbit Least Squares Program (IOLSQ) is coded for the Philco 2000 computer and is intended to be a part of the Semi-Automatic Program System (SPS) at the SPADATS Center in Colorado Springs, Colorado. This program operates in conjunction with the SPS executive routines.

IOLSQ is based on a theory employing a least-squares fit to the computed geocentric position vectors. The least squares procedure does not use the times of the observations; time enters into the formulation only as the epoch time, i.e., the time of the first observation. As the least squares method is not iterative, it involves only linear equations. The result is an initial orbit that is not severely affected by specific errors due to a few pieces of data. This is in contrast to an exact orbit based on only two position vectors.

Included in Section Two is the actual formulation used in IOLSQ to provide a detailed guide to the program operation. Section Three describes the input and output requirements while the flow charts are contained in Section Four. A summary of the experimentation that has been conducted with the program to evaluate the theory is included in Section Five.

For a complete discussion of the differential correction portion of IOLSQ, see Section 3 of Reference 2 listed at the end of this document. A discussion of the ephemeris part of differential correction can be found in Reference 1.

SECTION 2

FORMULATION

Computation for the Initial Orbit Least Squares Program involves the solution of a number of linear equations. These equations, solved in a straight-forward manner, yield the classical orbital elements, a, e, i, Ω, and ω. Each of the observations must initially be converted to position vectors, $\underline{r}_i$, in the inertial system thus requiring complete observational fixes of range and two angles for this procedure. The formulation follows:

(1) <u>Determination of the Geocentric Position Vector</u>, $\underline{r}_i$, <u>and the Unit Position Vector</u> $\underline{U}_i$:

$\underline{r}_i$ and $\underline{U}_i$ are calculated for each observation for $i = 1,2,3,\ldots,N$; where N is the number of acceptable observations

$\underline{r}_i = \rho_i \underline{L}_i - \underline{R}_i$, where ρ_i is the observed range.

$$\underline{U}_i = \frac{\underline{r}_i}{r_i}$$

The relationship between the satellite position, the dynamical center, and the observer is illustrated in Figure 1.

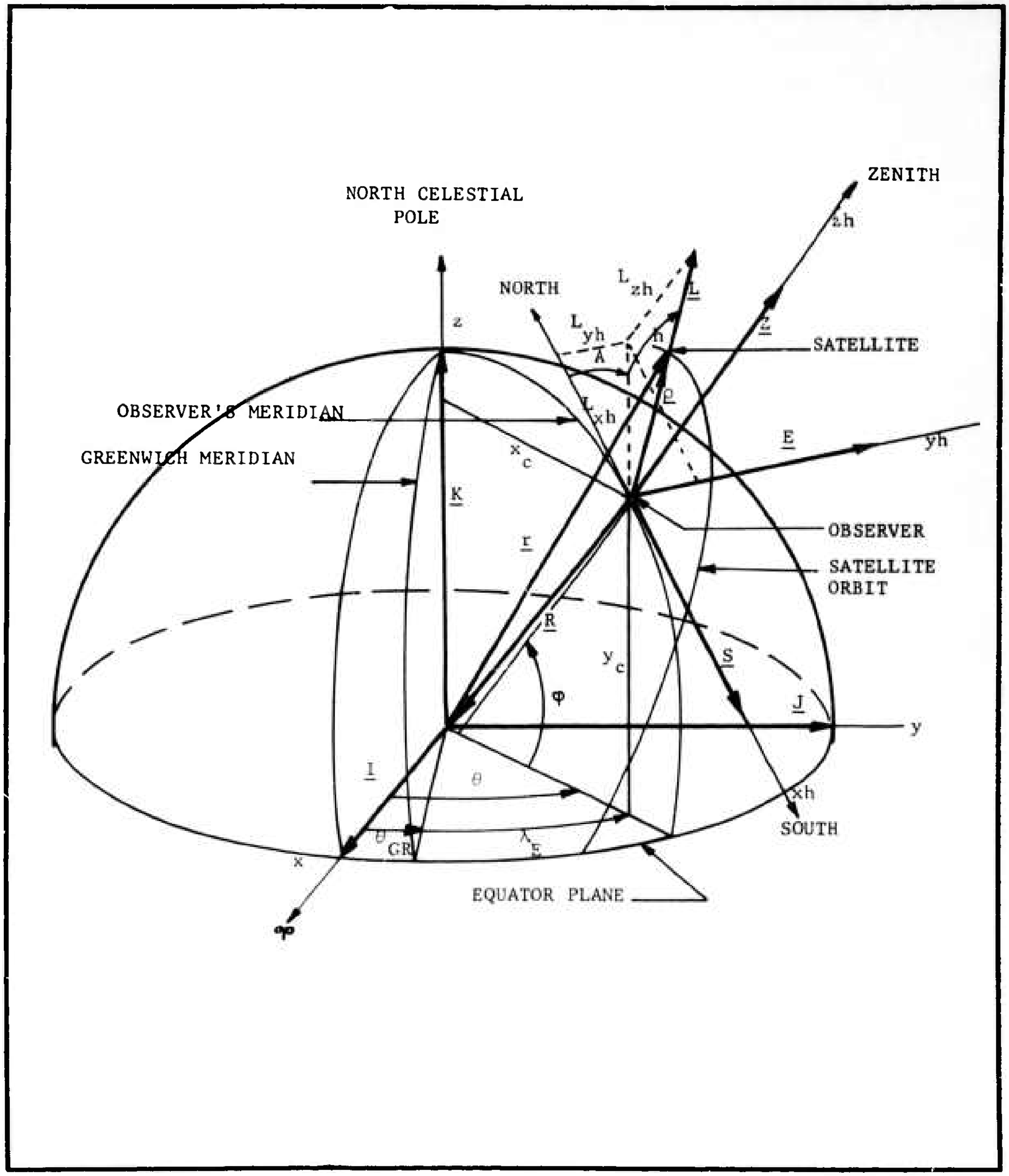


FIGURE 1. OBSERVATIONAL GEOMETRY

The position of the Station, $\underline{R}_i$, is calculated as follows:

$$X = \left[\frac{X}{\cos \theta}\right] \cos \theta$$

$$Y = \left[\frac{X}{\cos \theta}\right] \sin \theta$$

$$Z = Z$$

$$R = \sqrt{X^2 + Y^2 + Z^2}$$

where θ, the local sidereal time, is computed from

$$\theta = \theta_{gr} + \lambda_E$$

$$\theta_{gr} = \theta_{gr_o} + D(0.9856472) + F(360.9856472)$$

θ_{gr_o} = Greenwich sidereal time at the beginning of the year, and D and F are days and fractions of days, respectively, from the start of the epoch year to epoch.

ϕ is the station latitude (positive to the north)

λ_E is the longitude east

$\frac{X}{\cos \theta}$ is the given distance of the station from the polar axis and Z is the polar component of the station's geocentric position. These quantities are normally acquired in referencing the SPS system for station coordinates.

The unit vector, $\underline{L}_i$, is computed from the observed quantities azimuth (A) and elevation (h):

$$L_x = - \cos h \cos A \sin \phi \cos \theta - \cos h \sin A \sin \theta$$
$$+ \sin h \cos \phi \cos \theta$$

$$L_y = -\cos h \cos A \sin \phi \sin \theta + \cos h \sin A \cos \theta$$

$$+ \sin h \cos \phi \sin \theta$$

$$L_z = \cos h \cos A \cos \phi + \sin h \sin \phi$$

such that $L = \sqrt{L_x^2 + L_y^2 + L_z^2}$

(2) <u>Calculation of S_{xx}, S_{xy}, S_{yy}, S_{xz}, S_{yz}, S_{zz}, the Least Squares Quantities used in the Solution of $\underline{W}_o$:</u>

$$S_{xx} = \sum_{i=1}^{N} U_{x_i}^{2}$$

$$S_{xy} = \sum_{i=1}^{N} U_{x_i} U_{y_i}$$

$$S_{xz} = \sum_{i=1}^{N} U_{x_i} U_{z_i}$$

$$S_{yz} = \sum_{i=1}^{N} U_{y_i} U_{z_i}$$

$$S_{yy} = \sum_{i=1}^{N} U_{y_i}^{2}$$

$$S_{zz} = \sum_{i=1}^{N} U_{z_i}^{2}$$

(3) **Calculation of $\underline{W}_o$, the Unit Vector Perpendicular to the Orbit Plane:**

$$\underline{W}_o = \frac{\hat{\underline{W}}}{\hat{W}} \qquad \text{(See Figure 2.)}$$

where $\hat{W} = \sqrt{\hat{W}_x^2 + \hat{W}_y^2 + \hat{W}_z^2}$

and $$\hat{W}_x = \frac{S_{yz}\, S_{xy} - S_{xz}\, S_{yy}}{S_{xx}\, S_{yy} - S_{xy}^2}$$

$$\hat{W}_y = \frac{S_{xz}\, S_{xy} - S_{yz}\, S_{xx}}{S_{xx}\, S_{yy} - S_{xy}^2}$$

$$\hat{W}_z = 1.0$$

(4) **Calculation of $\underline{U}_o$ and $\underline{V}_o$, where $\underline{V}_o$ is the Unit Vector in the Orbit Plane Perpendicular to $\underline{U}_o$:**

$\underline{U}_o$ and $\underline{V}_o$ form an orthogonal system with $\underline{W}_o$

Let $n = \underline{U}_i \cdot \underline{W}_o$ (subscript 1 denotes the first observation)

Then $$\underline{U}_o = \frac{\hat{\underline{U}}_o}{\hat{U}_o}$$

where $\hat{\underline{U}}_o = \underline{U}_1 - n\underline{W}_o$

$$\hat{U}_o = \sqrt{\hat{U}_{x_o}^2 + \hat{U}_{y_o}^2 + \hat{U}_{z_o}^2}$$

and

$$\underline{V}_o = \underline{W}_o \times \underline{U}_o$$

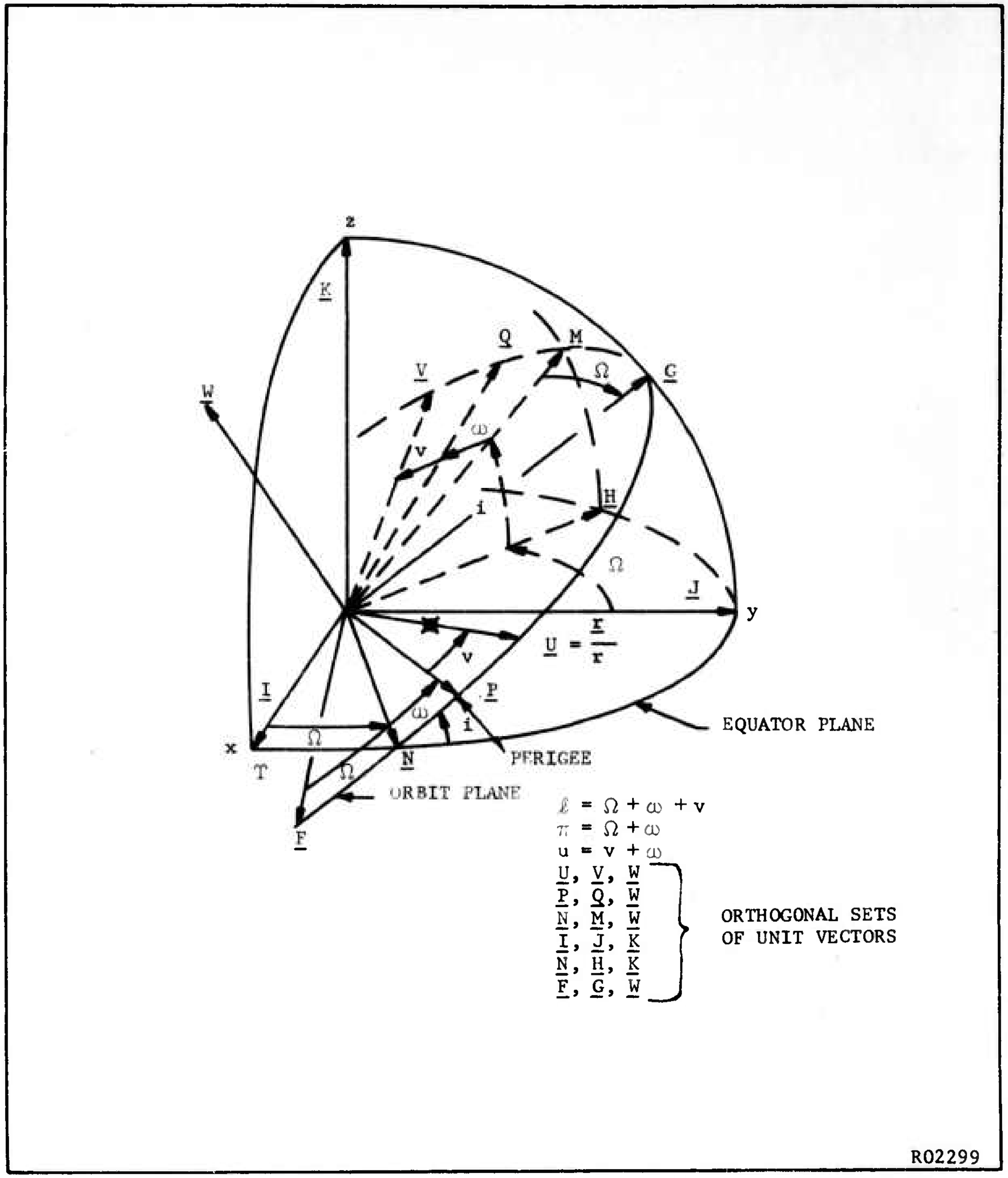


FIGURE 2. PROJECTION OF ORBIT-PLANE ON CELESTIAL SPHERE WITH ORIENTATION UNIT VECTORS AND ANGLES

Restricting W_{z_o} to a positive value will generate the wrong solution when the true orbit is retrograde and, in this case, the vector $\underline{V}_o$ will be calculated in the wrong direction. To avoid this problem, the direction of motion of the satellite is determined by examining a pair of vectors, $\underline{U}_i$ and $\underline{U}_{i+i}$, such that $0 < t_{i+1} - t_i \leq 40$ minutes. If W_{z_o} is selected correctly, then

$$\text{sign of } (\underline{U}_i \times \underline{U}_{i+1})_j = \text{sign of } \underline{W}_j$$

where j denotes the vector component with the largest magnitude.

If this sign equality does not occur, W_{z_o} has been chosen in the wrong direction. The signs of the components of $\underline{W}_o$ and $\underline{V}_o$ are then changed, and the calculations continue.

(5) <u>Computation of $\cos(v_i - v_o)$ and $\sin(v_i - v_o)$:</u>

$$\cos(v_i - v_o) = \underline{U}_i \cdot \underline{U}_o$$

$$\sin(v_i - v_o) = \underline{U}_i \cdot \underline{V}_o$$

(6) <u>Formulation of S_1, S_{11}, S_{12}, S_{13}, S_2, S_{22}, S_{23}, S_3, S_{33}, the Least Squares Quantities used in the Solution of p, e sin v_o and e cos v_o:</u>

$$S_1 = \sum_{i=1}^{N} \sin(v_i - v_o)$$

$$S_{11} = \sum_{i=1}^{N} \sin^2(v_i - v_o)$$

$$S_{12} = \sum_{i=1}^{N} \sin (v_i - v_o) \cos (v_i - v_o)$$

$$S_{13} = \sum_{i=1}^{N} \frac{1}{r_i} \sin (v_i - v_o)$$

$$S_2 = \sum_{i=1}^{N} \cos (v_i - v_o)$$

$$S_{22} = \sum_{l=i}^{N} \cos^2 (v_i - v_o)$$

$$S_{23} = \sum_{i=1}^{N} \frac{1}{r_i} \cos (v_i - v_o)$$

$$S_3 = \sum_{i=1}^{N} \frac{1}{r_i}$$

$$S_{33} = \sum_{i=1}^{N} \left(\frac{1}{r_i} \right)^2$$

(7) Calculation of p, e sin v_o, e cos v_o:

The following least squares equations are solved to obtain p, e sin v_o, e cos v_o.

$$S_1 = (e \sin v_o)\, S_{11} - (e \cos v_o)\, S_{12} + p\, S_{13}$$

$$S_2 = (e \sin v_o)\, S_{12} - (e \cos v_o)\, S_{22} + p\, S_{23}$$

$$S_3 = (e \sin v_o)\, S_{13} - (e \cos v_o)\, S_{23} + p\, S_{33}$$

(8) Determination of e, a, e cos E_o, e sin E_o:

$$e = \sqrt{(e \cos v_o)^2 + (e \sin v_o)^2}$$

$$a = \frac{p}{1-e^2}$$

$$e \cos E_o = \frac{e(e + \cos v_o)}{1 + e \cos v_o}$$

$$e \sin E_o = \frac{\sqrt{1-e^2}\,(e \sin v_o)}{1 + e \cos v_o}$$

$$E_o = \tan^{-1}\left[\frac{\sqrt{1-e^2}\,\sin v_o}{e + \cos v_o}\right] \qquad 0 \leq E_o < 2\pi$$

(9) **Calculation of $\underline{h}$, the Angular Momentum Vector Directed Along $\underline{W}_o$:**

$$\underline{h} = \sqrt{p}\ \underline{W}_o$$

(10) **Calculation of Ω and i, the Longitude of the Ascending Node and the Inclination of the Orbit Plane:**

Since the three equatorial component equations of $\underline{W}_o$ are

$$W_{x_o} = \sin \Omega \sin i$$

$$W_{y_o} = - \cos \Omega \sin i$$

$$W_{z_o} = \cos i$$

then $$\Omega = \tan^{-1}\left[\frac{W_{x_o}}{-W_{y_o}}\right] \qquad 0 \leq \Omega < 2\pi$$

and $$i = \tan^{-1}\left[\frac{\sqrt{1-W_{z_o}^{\,2}}}{W_{z_o}}\right] \qquad 0 \leq i < \frac{\pi}{2}$$

(11) **Calculation of $\underline{a}$, the Vector Directed to the Perifocus, and Its Components, a_{xN} and a_{yN}:**

a_{xN} is the component of $\underline{a}$ along $\underline{N}$, the unit vector directed to ascending node.

a_{yN} is the component of $\underline{a}$ along $\underline{M}$, the unit vector in the orbit plane perpendicular to $\underline{N}$, in the direction of motion.

$$\underline{a} = \underline{U}_o\, e \cos v_o - \underline{V}_o\, e \sin v_o$$

$$a_{xN} = \frac{a_y W_x - a_x W_y}{\sqrt{1 - W_z^2}}$$

$$a_{yN} = \frac{-W_z\, (a_x W_x + a_y W_y)}{\sqrt{1 - W_z^2}} + a_z \sqrt{1 - W_z^2}$$

(12) <u>Calculation of ω, the Argument of Perigee</u>:

$$\omega = \tan^{-1}\left(\frac{a_{yN}}{a_{xN}}\right) \qquad 0 \leq \omega < 2\pi$$

(13) <u>Calculation of $\dot{\Omega}, \dot{\omega}$, P_a, and H_q</u>:

$\dot{\Omega}$ is the rate of change of the longitude of ascending node.

$\dot{\omega}$ is the rate of change of the argument of perigee.

P_a is the anomalistic period.

H_q is the altitude of perigee in statute miles.

$$\dot{\Omega} = \frac{k_e W_{z_o}}{a^{3/2} p^2} \quad (1.62329 \times 10^{-3})$$

$$\dot{\omega} = \frac{k_e}{2\, a^{3/2}\, p^2} \quad (1.62329 \times 10^{-3}) \; (5\, W_{z_o}^{\;2} - 1)$$

$$P_a = \frac{2\pi}{k_e} \quad a^{3/2}$$

$$H_q = [a(1-e) - 1] \ (3963.199)$$

(14) <u>Calculation of the Mean Longitude at epoch, L_o:</u>

$$L_o = \pm \Omega + \omega + E_o - e \sin E_o \qquad \text{if } i \leq 90^\circ, \text{ use } + \Omega$$

$$\text{if } i > 90^\circ, \text{ use } - \Omega$$

SECTION 3

PROGRAM OPERATIONS

This program is executed in conjunction with the B-2 Semi-Automatic Program System (SPS) job schedule mode of operation. The normal operating procedures required for this mode of program execution are described in Reference 3.

Observations are read into memory from cards. Sensor coordinates are stored in core from cards or from the system SEAI file. The position vectors of the satellite are then calculated and stored in memory.

3.1 INPUT

IOLSQ has two input options:

0 : Observation Cards.

1 : Observation and Sensor Cards.

Required input data consists of observation types 2 or 3 and/or the Sensor Cards.* These data cards must be used in conjunction with the usual SPS schedule tape mode control cards.

---

*Observation Types: 2 - contains elevation, azimuth and range

3 - contains elevation, azimuth, range and range-rate.

The tape setup required for input is arranged as follows:

TAPE	DESCRIPTION
1	SPS Master Tape
2	Schedule Tape
4	SEAI Tape
11	Output Tape

The Schedule Tape used for input is generated from the following punched cards:

ID Card (70 SC TP)

JOB Card

REM Card

SPSJOB Card (IOLSQ)

Sensor Cards - optional

Observation Cards

END Card (END OF JOB)

END Card (ENDSCHED)

Blank Card.

The items shown in parentheses above are the code names to be used in the respective cards. These cards, along with the JOB, REM, Sensor and Observation Cards are described in Section 5.4 of Reference 2.

3.2 OUTPUT

All printed or punched output may be obtained from logical tape unit 11, using data select 1 for printed output and data select 2 for punched card output.

The printed output of IOLSQ begins with a heading and the date. This is followed by the N, M Elements and the Classical Elements at the time of the first observation (epoch time). The differential correction output follows, showing the observation residuals and correction data at each iteration followed by the final correction summary. Printout for the differential correction portion is exactly like that of all SPS differential correction packages. (See Reference 2.). In this standard differential correction output, the corrected elements are displayed, plus the corrected elements updated to the final revolution. For this program, the results of the two sets of new elements will be the same. See Figure 3, page 19.

The punched card output consists of two standard SPS 7-Card Element sets. System routines were used to obtain the punched output so that the two sets are duplicates. This is because the corrected set and **the updated set** are the same in IOLSQ.

3.3 PROGRAM NOTES

Comments may be printed indicating the operation of the IOLSQ Program. Any of the comments from the following list may be produced during the operation of the initial orbit portion of IOLSQ:

(1) "EXPONENT OVERFLOW AT THE $\genfrac{}{}{0pt}{}{\text{LEFT}}{\text{RIGHT}}$ HALF OF LOC. XXXXX"

(2) "SUBROUTINE ERROR AT THE $\genfrac{}{}{0pt}{}{\text{LEFT}}{\text{RIGHT}}$ HALF OF LOC. XXXXX"

(3) "NO COORDS SPECIFIED FOR SENSOR NO. XXXX"

This comment indicates that the OBSGET subroutine could not find from either the SEAI Tape or from the Sensor Cards the coordinates corresponding to the sensor number on one of the Observation Cards.

(4) "THERE ARE XXX OBSERVATIONS WHICH HAVE NO RANGE"

Only observations types 2 and 3 can be used by the initial orbit part of IOLSQ. This comment indicates the number of input observations that were rejected during the initial orbit determination.

(5) "NUMBER OF GOOD OBS. IS LESS THAN 5"

The least-squares theory in IOLSQ requires that there be a minimum of 5 acceptable observations in order that the calculations can proceed.

(6) "OBSERVATION BUFFER FULL, ONLY THE FIRST 150 TYPE 2 or 3 INPUT OBS. WILL BE USED"

This comment indicates that the maximum of 150 acceptable observations was exceeded. Therefore, calculations will proceed using only the first 150 type 2 and 3 observations .

(7) "THERE IS NO PAIR OF OBSERVATIONS WHICH ARE SEPARATED IN TIME BY MORE THAN 1 MIN. AND LESS THAN 40 MIN."

To determine the direction of motion of the satellite, the program attempts to examine two observations which are greater than 1 minute but less than 40 minutes apart. The above comment indicates that no such pair was found and the job is terminated.

The error comments that may result from the differential correction portion of the program are fully described in Section 3, pages 3-84 and 3-85, of Reference 2.

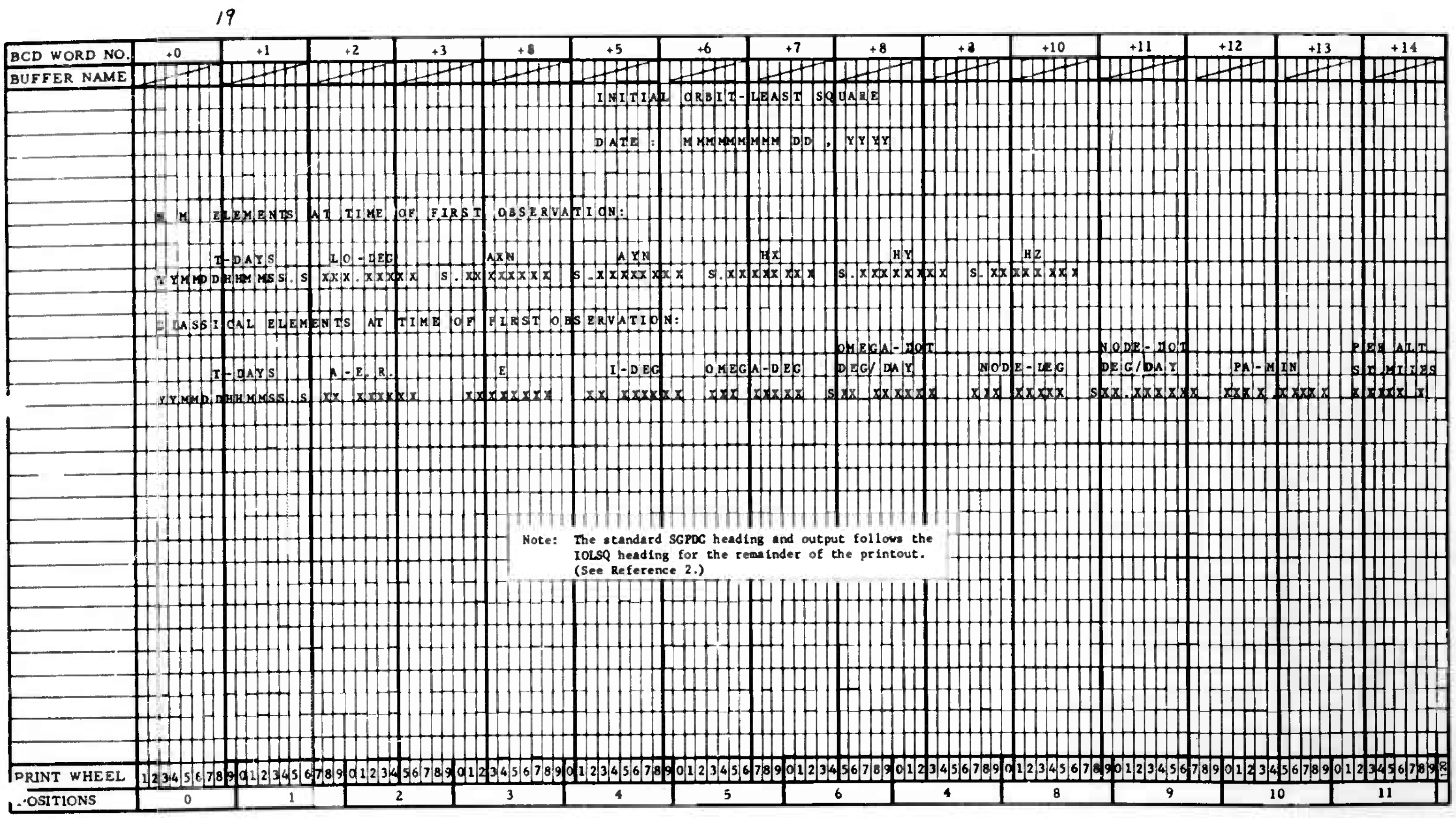


FIGURE 3. IOLSQ OUTPUT FORMAT

SECTION 4

FLOW DIAGRAMS

The following illustrations indicate the computational procedures of IOLSQ in diagrammatic form. Diamonds indicate decisions; arrows indicate the direction of logical flow; ovals indicate subroutines, and rectangles indicate mathematical or bookkeeping operations. The small circles indicate starts and exits and also serve as connectors from one page to the next. The oval at the entrance to a subroutine contains the name of that particular subroutine. A rectangle connected to a subroutine oval contains a brief description of that particular subroutine. Dotted lines from a subroutine oval indicate possible exits.

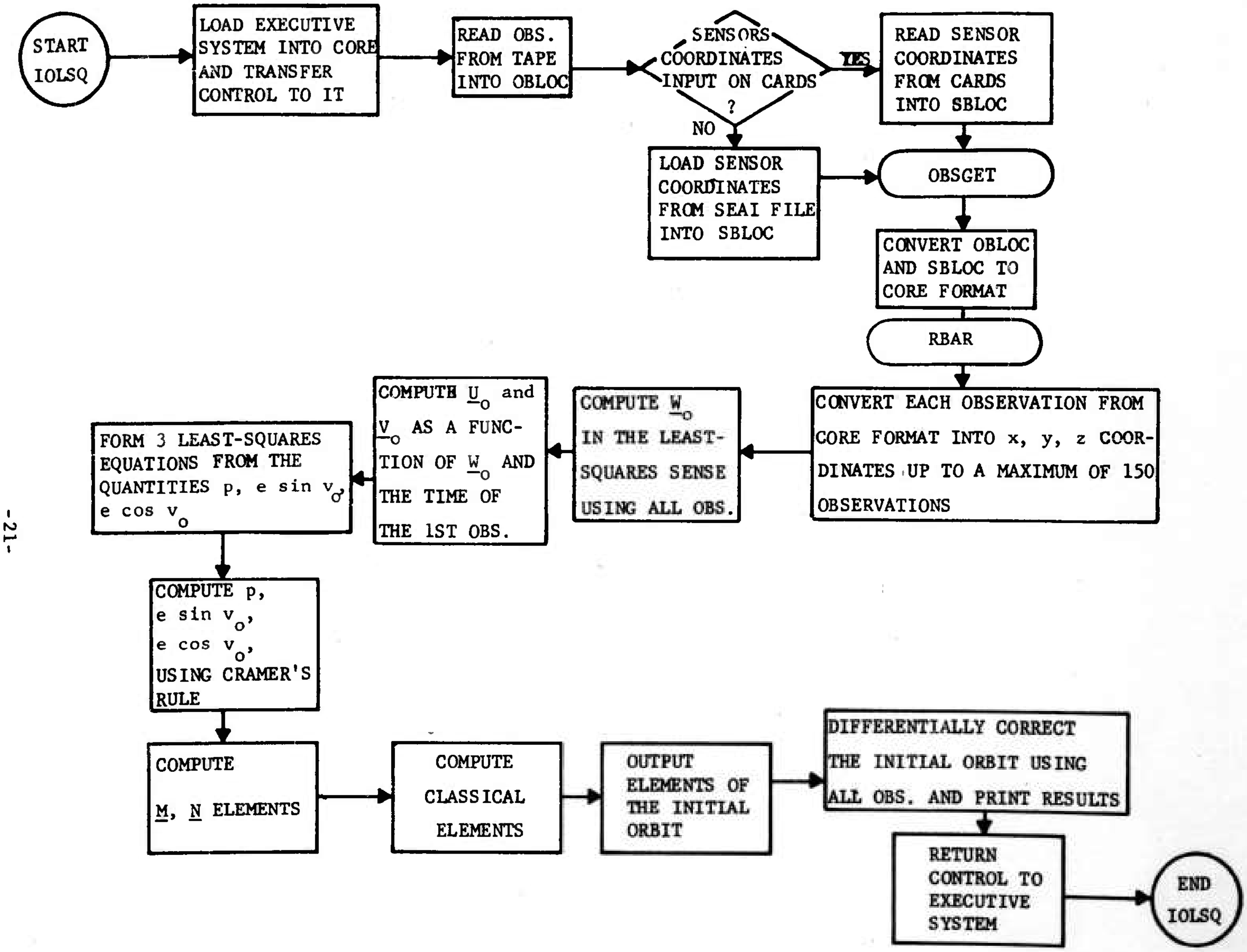


FIGURE 4. IOLSQ GENERAL FLOW DIAGRAM

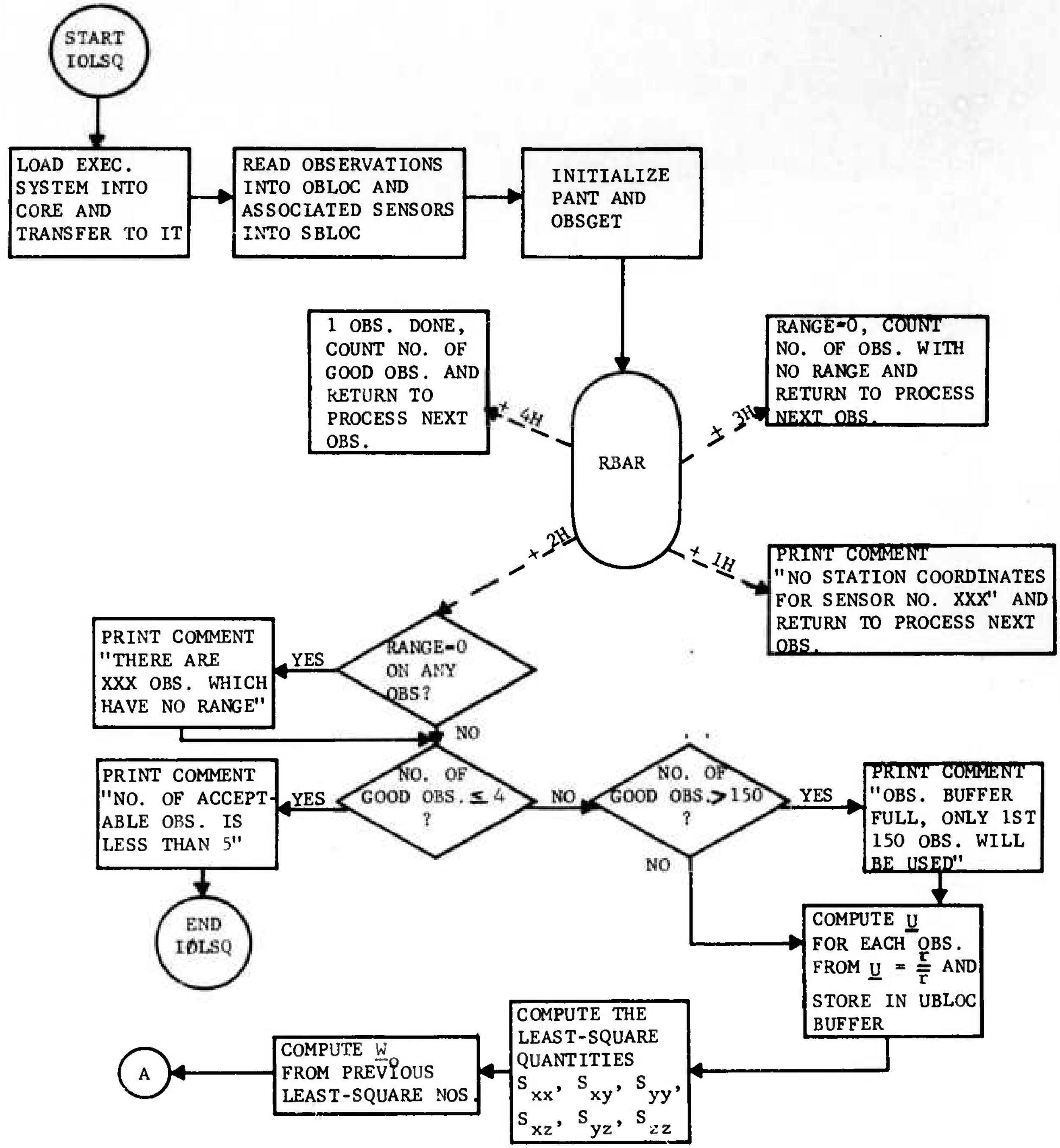


FIGURE 5. IOLSQ DETAILED FLOW DIAGRAM (1 OF 3)

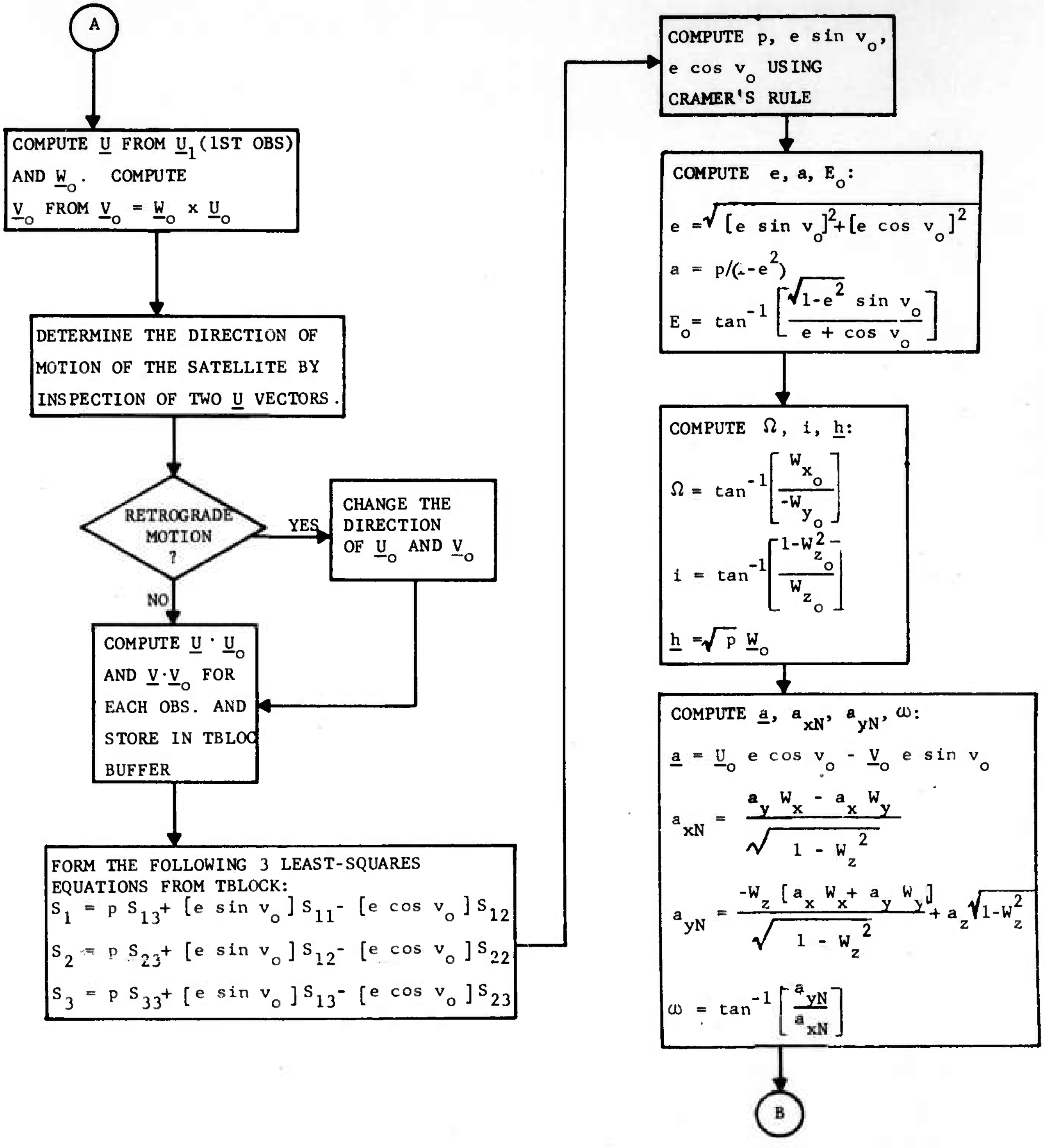


FIGURE 5. IOLSQ DETAILED FLOW DIAGRAM (2 OF 3)

B

COMPUTE $\dot{\Omega}, \dot{\omega}, P_A, H_q, L_o$:

$$\dot{\Omega} = \frac{k_e}{a^{3/2}\, p^2} (1.62329 \times 10^{-3})\, W_{z_o}$$

$$\dot{\omega} = \frac{k_e}{2a^{3/2}\, p^2} (1.62329 \times 10^{-3}) (5\, W_{z_o}^{\;2} - 1)$$

$$P_a = \frac{2\,\pi}{k_e}\, a^{3/2}$$

$$H_q = [\, a\,(1-e) - 1\,]\; 3963.199$$

$$L_o = E_o - e \sin E_o + \omega \pm \Omega$$

CONVERT
M, N AND CLASSICAL ELEMENTS TO UNITS FOR PRINTING AND PRINT-OUT INITIAL ORBIT

PUT M, N ELEMENTS IN MEMORY FOR D.C. USE

DIFFERENTIALLY CORRECT INITIAL ORBIT USING THE S.G.P,D.C.. PROCEDURE. ONLY THE 6 ORBITAL ELEMENTS WILL BE CORRECTED

END IOLSQ

FIGURE 5. IOLSQ DETAILED FLOW DIAGRAM (3 OF 3)

SECTION 5

PROGRAM EXPERIMENTATION

5.1 SUMMARY OF TESTING

In this section a summary of twenty cases that were run with the IOLSQ Program is presented to illustrate the capabilities of the initial orbit determination.

Observations were chosen from four different satellites, each satellite representing a different type of orbit. The following table summarizes the different orbits used.

TABLE I

SUMMARY OF TEST ORBITS

Satellite Number	Drag	Eccentricity
011	Low	Moderate
070	Moderate	Low
116	Low	Low
213	High	Low

Five runs were made for each satellite with each successive run having more observations than the previous run and covering a longer time span. The time span and the number of observations used are included in the following tables along with the initial and corrected elements. The test cases are presented here to illustrate the accuracy of the initial orbit portion of IOLSQ rather than the differential correction portion. Each initial orbit is compared with a reference orbit which

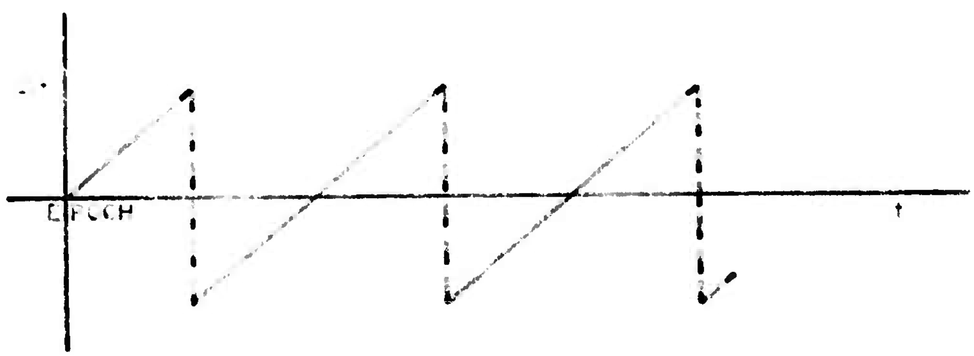
was obtained from the SGPDC Program. (See Reference 2.) The reference orbit was determined using file elements and the same observations as the initial orbit case with the largest number of observations.

5.2 RESULTS

Experimentation indicates that IOLSQ is quite adequate for determining an initial orbit when a group of associated observations is given. The initial orbital parameters are entered as input to the differential correction scheme, and the program appears to yield sufficiently accurate elements for this purpose.

In the early phase of the experimentation, the initial elements were not always sufficiently accurate when corrected over a long time span of observations. This led to divergence in the correction process. To mitigate this problem, a special period correction was included in IOLSQ to improve the semi-major axis. This correction is executed by fitting a straight line in the least-squares sense to the time residuals. The correction to the semi-major axis is then computed from the slope of this line.

When the time span covered by the observations is long, the time residuals may exhibit a "saw-tooth" effect. The time residual is similar to the angle in the orbit plane between the computed position and the observed position. The time residual is thus computed from the angle between two vectors and necessarily lies between $-\pi$ and $+\pi$. Therefore, when the time residuals exceed one-half the period, they will change sign and exhibit the saw-tooth effect. The sketch below illustrates Δt as a function of time:



To fit, in a least-squares sense, a line to the time residuals, it becomes necessary to eliminate the "saw-tooth", i.e., Δt must be made a linear function of time. This is accomplished by adding or subtracting the value of the period when two successive time residuals differ by more than one-half the period. This special period correction is invalid in the case where two successive time residuals are far enough apart that there exists more than one change of sign between them.

IOLSQ differs from other initial orbit programs in that the semi-major axis is determined from geometrical rather than dynamical considerations. Thus, it is to be expected that there will be an error in the semi-major axis when the points in space, as computed from the observations, do not uniquely define an ellipse. The experimentation has shown that the semi-major axis is, indeed, inadequately determined when the observations are concentrated at one point in the orbit but becomes increasingly better as more observations are added. This effect is illustrated in the 3-hour correction of Satellite 011 (see Table II).

The initial orbit formulation assumes that the longitude of the ascending node, Ω, is constant. It can, therefore, be expected that due to the precession of the node, Ω will be poorly determined when the span of the observations is large. This limitation of the program is illustrated by Satellites 070 and 213 (see Tables III and V) where Ω becomes increasingly worse and is in error by as much as 16° when the observation span is 16 days and 15 hours.

Another limitation of the program is illustrated by Satellites 070, 116 (see Table IV), and 213. The argument of perigee, ω, is poorly defined when the eccentricity is low. The large errors in ω are due to the low eccentricity and should be corrected by the differential correction. For low eccentricities, however, a large error in ω does not necessarily indicate an inaccurate orbit.

The eccentricity and the argument of perigee are poorly defined when the observations represent a single point in space. In this case, the program cannot uniquely determine an ellipse to fit the observations. This situation is illustrated by Satellite 011, Case 5. The observations are concentrated at a single point in space, and the differential correction could not adequately determine an orbit except when the correction of the eccentricity and argument of perigee was inhibited.

The effect of high drag is illustrated by Satellite 213. The computation of drag is not included in IOLSQ, and consequently the differential correction cannot always converge to an orbit that closely fits the observations. Cases 3, 4 and 5 could converge to an

orbit only when the correction of the eccentricity was inhibited. When the eccentricity was corrected, the differential correction began to diverge steadily. These cases represent the best results that can be obtained using the usual "no-drag" assumption.

The effect of bad observations is illustrated by the cases run on Satellite 070. Cases 3, 4 and 5 contain a group of observations that represent a single point in space. This point is not compatible with the rest of the orbit, and the differential correction could not determine an orbit to fit all of the observations except when the eccentricity was not corrected. Cases 4 and 5 show that the effect of these bad observations is minimized when observations are added that more clearly define the orbit.

As a result of the testing (and in view of the foregoing comments), it is concluded that IOLSQ is best suited for orbit determinations using groups of observations that cover a time span on the order of four or five days. Enough observations must be available for input to enable the program to geometrically determine the semi-major axis. The time span must be small enough so that the precession of Ω is not a dominating factor. The advantage of IOLSQ is that its accuracy is not severely affected by errors in a few observations as would occur for other initial orbit programs* based on one- or two-position fixes.

5.3 EXAMPLE OF PROGRAM OPERATION

Portions of an IOLSQ test case are presented on the following pages to illustrate the combined operation of the initial orbit determination and the differential correction procedure. The test case is reproduced in the actual computer output format for Satellite 116, Case 3. (See Figure 7.) Case 5 printed at the end of the output indicates the fifth differential correction iteration for Case 3. Due to space limitations, the test case output is abbreviated to show the initial elements, the last pass residuals, and the final element summary. All input data for this case are shown in the reproduced 80-80 listing of the input deck (see Figure 6).

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*Such as Initial Orbit Radar Fix (IORF) Program, which computes the initial orbit from 2 or 3 radar fixes, and Initial Orbit Radar Track (IOHG) Program, which calculates the initial orbit from a radar track.

TABLE II

IOLSQ INITIAL AND CORRECTED ELEMENT SETS FOR SATELLITE 011

		a (E.R.)	e	i (deg)	Ω (deg)	ω (deg)	P_A (min)
SGP REFERENCE ORBIT		1.302053	0.16459	32.788	110.728	78.091	125.460
CASE 1 3 Hours 26 Observations	INITIAL	1.230400	0.12455	32.598	111.784	64.866	115.313
	CORRECTED	1.2256795*	0.11639	32.634	109.895	72.190	114.685
CASE 2 1 Day + 2 Hours 48 Observations	INITIAL	1.284587	0.15418	32.704	107.941	82.609	123.013
	CORRECTED	1.3013319	0.16464	32.647	111.062	78.523	125.461
CASE 3 2 Days + 1 Hour 70 Observations	INITIAL	1.295298	0.16194	32.564	108.238	82.192	124.555
	CORRECTED	1.3013288	0.16260	32.696	110.988	81.797	125.461
CASE 4 3 Days + 2 Hours 96 Observations	INITIAL	1.292486	0.15988	32.698	10[illegible].144	83.307	124.150
	CORRECTED	1.3013571	0.16278	32.703	111.107	82.167	125.465
CASE 5 5 Days + 2 Hours 108 Observations	INITIAL	1.324761	0.17994	32.808	104.595	87.558	128.829
	CORRECTED	1.3013054	0.17994**	32.867	109.051	87.469**	125.457

*All observations except one within 3 min. of one another, hence the poor semi-major axis.

**e and ω not corrected in the differential correction process.

TABLE III

IOLSQ INITIAL AND CORRECTED ELEMENT SETS FOR SATELLITE 070

		a (E.R.)	e	i (deg)	Ω (deg)	ω(deg)	P_A (min)
SGP REFERENCE ORBIT		1.0797636	0.00584	97.347	324.999	223.992	94.860
CASE 1 5 Days + 5 Hours 27 OBSERVATIONS	INITIAL	1.074933	0.01140	99.454	328.325	245.179	94.163
	CORRECTED	1.0804762	0.00587	97.297	324.978	233.644	94.860
CASE 2 8 Days + 21 Hours 56 OBSERVATIONS	INITIAL	1.075284	0.01091	102.152	332.855	244.967	94.209
	CORRECTED	1.0804714	0.00584	97.446	324.997	230.667	94.860
CASE 3 13 Days + 11 Hours 93 OBSERVATIONS	INITIAL	1.073103	0.02721	110.136	338.605	287.341	93.922
	CORRECTED	1.0804414	0.02721*	97.055	324.892	287.361*	94.855
CASE 4 15 Days + 15 Hours 106 OBSERVATIONS	INITIAL	1.075303	0.02107	109.939	339.379	286.639	94.211
	CORRECTED	1.0804734	0.00393	97.402	325.003	239.468	94.860
CASE 5 16 Days + 15 Hours 125 OBSERVATIONS	INITIAL	1.077235	0.01601	109.598	340.7799	289.017	94.465
	CORRECTED	1.0806248	0.00404	97.467	325.023	145.955	94.880

*e and ω not corrected in the differential correction process.

TABLE IV

IOLSQ INITIAL AND CORRECTED ELEMENT SETS FOR SATELLITE 116

		a (E.R.)	e	i (deg)	Ω (deg)	ω (deg)	P_A (min)
SGP REFERENCE ORBIT		1.146960	0.00768	66.799	93.437	137.544	103.815
CASE 1	INITIAL	1.151539	0.01356	67.098	92.232	110.150	104.406
17 Hours 20 OBSERVATIONS	CORRECTED	1.1473379	0.00762	66.799	93.403	142.030	103.817
CASE 2	INITIAL	1.51883	0.01326	66.834	89.937	119.695	104.453
2 Days + 9 Hours 71 OBSERVATIONS	CORRECTED	1.1473398	0.00721	66.804	93.431	139.730	103.818
CASE 3	INITIAL	1.150085	0.01114	67.273	87.723	123.966	104.208
4 Days + 7 Hours 108 OBSERVATIONS	CORRECTED	1.147339	0.00746	66.802	93.429	142.088	103.818
CASE 4	INITIAL	1.128985	0.01348	66.758	86.177	285.040	101.354
5 Days + 2 Hours 150 OBSERVATIONS	CORRECTED	1.1473421	0.00633	66.749	93.541	127.749	103.818
CASE 5	INITIAL	1.131450	0.01064	66.444	85.453	283.166	101.686
6 Days + 1 Hour 150 OBSERVATIONS	CORRECTED	1.147341	0.00706	66.799	93.437	140.046	103.818

TABLE V

IOLSQ INITIAL AND CORRECTED ELEMENT SETS FOR SATELLITE 213

		a (E.R.)	e	i (deg)	Ω (deg)	ω (deg)	P_A (min)
SGP REFERENCE ORBIT		1.046857	0.01190	81.206	28.578	304.681	90.560
CASE 1	INITIAL	1.060330	0.00332	80.907	27.667	82.906	92.251
2 Days + 4 Hours 18 OBSERVATIONS	CORRECTED	1.0472239	0.01185	81.166	28.150	291.194	90.514
CASE 2	INITIAL	1.060313	0.00378	81.094	26.515	85.832	92.248
5 Days + 4 Hours 40 OBSERVATIONS	CORRECTED	1.0469900	0.01357	81.191	27.996	285.700	90.483
CASE 3	INITIAL	1.062472	0.00478	78.588	22.222	84.297	92.530
9 Days + 4 Hours 75 OBSERVATIONS	CORRECTED	1.0464638	0.00478*	81.259	29.484	84.332*	90.415
CASE 4	INITIAL	1.060639	0.00342	78.246	18.317	106.177	92.291
11 Days + 4 Hours 94 OBSERVATIONS	CORRECTED	1.0463125	0.00342*	81.250	29.699	106.209*	90.396
CASE 5	INITIAL	1.059583	0.00318	78.363	16.989	121.483	92.153
13 Days + 15 Hours 110 OBSERVATIONS	CORRECTED	1.0460965	0.00318*	81.567	28.786	121.513*	90.367

*e and ω not corrected in the differential correction process.

```
70 SCHTPR
                  JOB       IOLSQ SAT. 116  3
                  REM       INITIAL ORBIT LEAST SQUARE
                  REM       WITH DIFFERENTIAL CORRECTION
SPSJOB  IOLSQ     00
11625 216330712231013690047000 00960000305045-107117850             0000 U
116220039030714030512000141000087500000226129                            U
11621 039030713043520000887000 1274000008895 6                      0000 U
11621 039030713175009000550000 14860000115643                       0000 U
11621 039030713175225000532000 06860000117867                       0000 U
11625 03903071404483252345080 28331000116343-232953530              0000 U
11625 03903071404484452348290 27780000114549-194961900              0000 U
11625 03903071404485652350580 27098000112346-145665450              0000 U
11625 03903071404505652342710 20953000123955 314218540              0000 U
11625 03903071404510252341630 20799000126671 328414430              0000 U
11625 03903071404512052338670 20260000132544 376024430              0000 U
11625 03903071404513252336710 19940000137502 405213110              0000 U
11625 03903071404514452333900 19685000142593 426840540              0000 U
11625 03903071404515052333640 19522000144993 438979330              0000 U
11625 03903071404520253931700 19351000150347 459717200              0000 U
11625 03903071404521453930470 19114000155942 478601810              0000 U
11625 03903071404522653928450 18964000162347 495021610              0000 U
11625 03903071404523854625810 18870000168129 507809030              0000 U
11625 03903071404525054623020 18839000170979 519632770              0000 U
11625 03903071404525654622850 18749000176628 524228830              0000 U
11625003903071418033252346560021755000141386M26745900                    U
116220039030714180917000255000017100000181594                            U
11625 03903071503142652321880 01654000186378-399375380              0000 U
11625 03903071503143252321970 01757000184059-391406400              0000 U
116220039030715050220000247000277200000172236                            U
11625 03903071505063252318520 21353000196841 387588710              0000 U
11625 03903071505064452319200 20926000199817 406788380              0000 U
11625 03903071505065052318190 20966000201902 414108710              0000 U
11625 03903071505070252317460 20928000209132 428953250              0000 U
11625 03903071505070852317350 20801000209882 432011110              0000 U
11625 03903071716591452333640 18322000156333-500933480              0000 U
11625 03903071716592052334690 18217000152337-493335150              0000 U
11625 03903071716593252337150 18048000146394-476470580              0000 U
11625 03903071716593852338050 18101000143455-467760300              0000 U
11625 03903071716595052340340 17879000138662-449227800              0000 U
11625 03903071717012051561760 15409000107607-206266730              0000 U
11625 03903071717013252364310 14209000105291-161974050              0000 U
11625 03903071717013852365430 13656000104974-137140500              0000 U
11625 03903071717015052366770 12770000101936-088770680              0000 U
11625 03903071717015652367030 12106000101389-079319100              0000 U
11625 21633071717135204110150 01454000260917 549840740              0008 U
11625 21633071717102241329040 34079000163186 332176530              0000 U
11625 21633071715274701607420 04269000284459 576027170              0000 U
11625 21633071715260719315680 04959000228193 524488280              0000 U
116220216330717034503172 507002555400 294091  6934556                    U   08488 1
11625 21633071702041795811400 18962000244006 609793380              0000 U
11625 21633071701561211331460 30999000148547-491278040              0000 U
11625 21633071700181956303620 11063000309912 544410720              0000 U
11625 21633071700122021924520 05598000172847 083340650              0000 U
11625 21633071622262786801690 05203000328238 268257940              0000 U
11625 21633071622252797303360 04489000314055 207230420              0000 U
11625 21633071603330055809440 23132000258717 275059370              0000 U
11625 21633071603272116008650 28768000264952-304878160              0000 U
11625 21633071603240173613750 23710000227431 203116200              0000 U
11625 21633071603182253708190 29643000267785-399523640              0000 U
```

FIGURE 6. SAMPLE INPUT DATA - SATELLITE 116, CASE 3 (1 OF 2)

```
11625 21633071601505964407330 15172000276264 631772930          0000 U
11625 21633071600020538805770 09068000289513 448097320          0000 U
11625 21633071523590570614080 06615000225692 240588920          0000 U
11625 21633071523523669408280 35709000270465-408623090          0000 U
11625 21633071522101588001560 03715000333487 201948650          0000 U
11625 21633071522072322203330 01533000315336 002094170          0000 U
11625 21633071503225215502480 19371000321218 523487530          0000 U
11625 21633071503160321322280 25740000181935-036119840          0000 U
11625 21633071501374347201970 13807000325355 615260470          0000 U
11625 21633071501322403633770 11623000141831 445910480          0000 U
11625 21633071423411455011290 02759000246912-075575540          0000 U
11625 21633071423385479708040 00591000272296-278765870          0000 U
11625 21633071423375490105950 35811000290918-343852010          0000 U
11625 21633071420013410005860 34094000296870 034266590          0000 U
11625 21633071418133046324080 33561000181828 212252720          0000 U
11625 21633071418072751113880 25949000247642-476841230          0000 U
11625 21633071418054768706560 25055000296763-527138430          0000 U
11625 21633071414375891005600 10951000305638-125798610          0000 U
11625 21633071414362906903970 12084000322084-235696340          0000 U
11625 21633071403032362033800 22340000141403 232212230          0000 U
11625 21633071402593118024700 29710000171611-428100750          0000 U
11625 21633071401161228033700 06640000142144 129356850          0000 U
11625 21633071401122551021600 00160000186252-439590900          0000 U
11625 21633071323272740007500 03350000276320 048184500          0000 U
11625 21633071323253354007100 01640000279841-115642800          0000 U
11625 21633071317593354029900 35030000160862 361198430          0000 U
11625 21633071317555574035700 27680000145295-250374080          0000 U
11625 21633071316170340002600 04290000295668 570430350          0000 U
11625 21633071316125380026600 07070000173390 345445800          0000 U
11625 21633071304350461805710 24730000289795 147166580          0000 U
11625 21633071304343467106080 25154000286108 103633740          0000 U
11625 21633071304333477706480 26033000282610 011564280          0000 U
11625 21633071302495895439500 18239000128023 438108300          0000 U
11625 21633071302424971113550 32337000229981-609608060          0000 U
11625 21633071301053024809580 09861000257567 473338580          0000 U
11625 21633071300591091720690 02291000189782-224539770          0000 U
11625 21633071223104251004800 01360000302636-071350130          0000 U
11625 21633071223113921004600 02030000302080-002038580          0000 U
116250328 30717134507608 320681080591 326555-22621643                U    07792 1
116250328 30717135026981 32488 661678 319982 20198542                U    07806 1
116250328 30717153557433 68472 750573 280539 35189224                U    08012 2
116250329 30716235002989 324882403108 324747-18289677                U    04975 1
116250329 30717013802746 684722725396 291689-18057828                U    05167 1
116250329 30717032409866 324882926862 325859-18479371                U    05460 1
116250329 30717032425686 324882927876 322062-18379255                U    05463 1
116250329 30717032518235 684722929904 286967-18258061                U    05466 1
116250329 30717051110296 324883035026 318636-17968251                U    05776 1
116250329 30717051203422 684723018345 285392-17757479                U    05779 1
116250329 30717065916295 684722977989 285300-13352360                U    06038 1
ENDOFJOBR
ENDSCHEDR
```

112 CC

FIGURE 6. SAMPLE INPUT DATA - SATELLITE 116, CASE 3 (2 OF 2)

```
BEGIN SCHED. TAPE
            JOB      TOLSQ SAT. 116  3
            REM      INITIAL ORBIT LEAST SQUARE
            REM      WITH DIFFERENTIAL CORRECTION
SPS JOB  TOLSQ

                                         START   TOLSQ
                                         12-19-21-37.9

                              INITIAL ORBIT-LEAST SQUARE

                              DATE :  DECEMBER 19 , 1963
```

N.M ELEMENTS AT TIME OF FIRST OBSERVATION:

T-DAYS	LO-DEG	AXN	AYN	HX	HY	HZ
6307122311313.6	192.45355	-.0 622468	.00224029	.98830829	-.33929113	.41430088

CLASSICAL ELEMENTS AT TIME OF FIRST OBSERVATION:

T-DAYS	A-E.R.	E	I-DEG	OMEGA-DEG	OMEGA-DOT DEG/DAY	NODE-DEG	NODE-DOT DEG/DAY	PA-MIN	PER.ALT. ST.MILES
6307122311313.6	1.150005	.01114134	67.272524	123.94599	-.774571	87.72336	-2.359309	104.20820	544.0

FIGURE 7. SAMPLE OUTPUT DATA - SATELLITE 116, CASE 3 (1 OF 5)

SIMPLIFIED GENERAL PERTURBATION DIFFERENTIAL CORRECTION DECEMBER 19, 1963 PAGE 25

SATELLITE NO. SATELLITE NAME: ELEMENT SET NO. 0 TIME OF EPOCH 193.9449674

TAG NO.	STA NO.	Y	MM	DD	TIME HH	MM	SS.SS	R A	R J	RANGE RES. KM.	RT ASCEN RES. KM.	DECL. RES. KM.	AZIMUTH RES. KM.	ELEV. RES. KM.	RR. RES. KM./SEC.	VECTOR MAG. KM.	DELTA T MIN.	U DEG.	BETA DEG.
116	216	3	07	12	23	10	13.69			-.4522+1			-.1320+2	.9040+1	.7029-2	.172+2	-.02	102	-.0
116	039	3	07	14	3	5	12.10		•	.1014+2			.3233+2•	-.3659+1		.355+2	.07	150	-.0
116	039	3	07	13	4	35	19.90		•	.1520+1			-.3040+1	.2273+2•		.231+2	-.04	149	-.0
116	039	3	07	13	17	50	8.98		•	-.4701+1			-.4132+2•	.3320+2•		.534+2	.11	26	.0
116	039	3	07	13	17	52	24.98		•	.1837+1			.3450+2•	.3281+2•		.476+2	-.09	33	-.0
116	039	3	07	14	4	48	32.51			-.1268+2			.1810+2	-.1123+2	.2958-1	.278+2	-.03	148	.0
116	039	3	07	14	4	48	44.52			-.4859+1			.1602+2	.1998+2	-.1902-1	.265+2	-.00	149	.1
116	039	3	07	14	4	48	56.51		•	-.6466+1			.4359+1	.3706+2•	.1939-1	.384+2	.02	149	.2
116	039	3	07	14	4	50	56.52			-.1870+1			.6644+1	.1850+2	.3000-1	.199+2	-.03	156	.0
116	039	3	07	14	4	51	2.51			.6102+1			.1394+2	.1700+2	.9749-3	.236+2	-.03	156	-.0
116	039	3	07	14	4	51	20.52			.1404+1			.1226+2	.1970+2	.1083-1	.232+2	-.04	157	.0
116	039	3	07	14	4	51	32.51			.4352+1			.8741+1	.2144+2	.2992-1	.239+2	-.03	158	.0
116	039	3	07	14	4	51	44.52			.5456+1			.1120+2	.1236+1	.9220-3	.136+2	-.01	159	-.0
116	039	3	07	14	4	51	50.51			.353 +1			.2558+1	.1893+2	.9676-2	.197+2	-.02	139	.0
116	039	3	07	14	4	52	2.52			.319 +1			.1408+2	.1783+2	.9586-2	.231+2	-.03	160	-.0
116	039	3	07	14	4	52	14.53		•	.2007+1			.5637+1	.3407+2•	.1329-1	.347+2	-.04	161	.0
116	039	3	07	14	4	52	26.52		•	.8725+1			.1276+2	.2701+2•	.1269-1	.323+2	-.03	161	-.0
116	039	3	07	14	4	52	38.53		•	.6342+1			.3376+2•	-.931300	-.6566-2	.320+2	-.02	162	-.2
116	039	3	07	14	4	52	50.54		•	-.2702+2•			.6366+2•	-.3501+2•	-.1946-1	.834+2	-.07	163	-.4
116	039	3	07	14	4	52	56.53		•	-.1087+1			.5869+2•	-.2035+2	-.3352-1	.621+2	-.04	163	-.4
116	039	3	07	14	18	3	32.50		•	.1428+3•			.1438+2	.1833+2	-.3841+2•	.203+3	-.14	24	.2
116	039	3	07	14	18	9	16.90		•	.4026+2•			-.7669+1	-.9398+1		.582+2	.07	44	.1
116	039	3	07	15	3	14	26.51			-.5571+1			.2211+2	.2046+2	.3494-1	.311+2	.06	134	.0
116	039	3	07	15	3	14	32.51			-.4878+1			.2073+2	.6432+1	.2404-1	.227+2	.04	135	.0
116	039	3	07	15	5	2	19.92			.1556+2			.1449+2	-.8430+1		.276+2	-.04	148	-.0
116	039	3	07	15	5	6	32.51		•	.5246+1			.3015+2•	-.2619+2•	.1048-1	.406+2	-.03	163	-.2
116	039	3	07	15	5	6	44.50		•	-.1242+2			-.4474+2•	.2893+2•	.2652-1	.561+2	.04	164	.3
116	039	3	07	15	5	6	50.52			-.1613+2			.43721a	.9855+1	.1635-1	.248+2	-.02	164	.1
116	039	3	07	15	5	7	2.51		•	.5761+1			.5071+2•	.1548+2	.7393-2	.539+2	-.09	164	-.2
116	039	3	07	15	5	7	8.52		•	-.1274+2			.3752+2•	.2950+2•	-.3558-1	.510+2	-.10	165	-.0
116	039	3	07	17	16	59	14.51			.1405+2			-.8379+1	.9433+1	-.2109-1	.235+2	-.00	19	-.0
116	039	3	07	17	16	59	20.51			.377 +1			-.1835+2	.9149+1	-.2192-1	.211+2	.02	20	-.1
116	039	3	07	17	16	59	32.51		•	.2356+1			-.2540+2•	.1527+2	-.1873-1	.298+2	.04	20	-.1
116	039	3	07	17	16	59	38.51			.1145+1			.5480-1	.4925+1	-.2258-1	.907+1	.01	21	.0
116	039	3	07	17	16	59	50.52			.7946+1			-.1152+2	.5819+1	-.3684-1	.171+2	.00	21	-.0
116	039	3	07	17	17	1	20.50			.166 +1			.1848+2	.8314+1	-.6429-1	.204+2	-.02	26	.1
116	039	3	07	17	17	1	32.51		•	-.3035+2			-.1538+2	.8245+1	-.9230-1•	.174+2	.03	27	-.0
116	039	3	07	17	17	1	38.52		•	.4085+1			-.2111+2	.8226+1	-.8481-1•	.237+2	.04	28	-.0
116	039	3	07	17	17	1	50.51		•	-.130 +2			-.6578+1	.1738+1	-.108754•	.196+2	.01	28	.0
116	039	3	07	17	17	1	56.51		•	-.1457+2			-.1149+2	-.3751+1	-.271893•	.238+2	.02	29	.0
116	216	3	07	17	17	13	52.12			-.1117+2			-.6164+1	.2989+1	.7698-2	.172+2	-.02	70	.0
116	216	3	07	17	17	10	22.40			-.8048+1			-.9080+1	-.504491	-.1398-1	.145+2	-.02	58	.0
116	216	3	07	17	15	27	47.00			-.1151+2			.1128+2	.4181+1	.6862-2	.202+2	-.03	62	-.0
116	216	3	07	17	15	26	7.14		•	-.2122+2			.5333+1	.3047+2•	-.2317-1	.431+2	-.07	56	.0
116	216	3	07	17	3	45	3.15	1		-.7547+1			-.8380+1	-.8103+1	.1437-1	.158+2	.01	146	.0
116	216	3	07	17	2	4	17.94			.22338			-.776169	.109353	.2268-1	.874+0	.00	156	.0
116	216	3	07	17	1	56	12.10			-.8374+1			-.2339+1	-.276981	-.9227-2	.120+2	.01	128	.0
116	216	3	07	17		18	19.57			-.5129+1			.3433+1	.1712+1	.1578-1	.819+1	-.00	149	-.0

FIGURE 7. SAMPLE OUTPUT DATA - SATELLITE 116, CASE 3 (2 OF 5)

SIMPLIFIED GENERAL PERTURBATION DIFFERENTIAL CORRECTION   DECEMBER 19, 1963   PAGE 26

SATELLITE NO.   SATELLITE NAME,   ELEMENT SET NO. 0   TIME OF EPOCH 193.9449674

TAG NO.	STA NO.	Y	MM	DD	TIME HH	MM	SS.SS	R A	R J	RANGE RES. KM.	RT ASCEN RES. KM.	DECL. RES. KM.	AZIMUTH RES. KM.	ELEV. RES. KM.	RR. RES. KM./SEC.	VECTOR MAG. KM.	DELTA T MIN.	U DEG.	BETA DEG.
116	210	3	07	17		12	20.22			-.5426+1			-.582808	.485679	.5376-2	.798+1	-.00	128	-.0
116	216	3	07	16	22	26	27.67			-.7068+1			-.3727+1	-.1388+2	-.1332-1	.176+2	-.00	121	.0
116	215	3	07	16	22	25	27.96			-.5834+1			-.3034+1	-.572080	-.3276-2	.880+1	-.01	118	-.0
116	210	3	07	16	3	33	.55			-.3470+1			-.5353+1	.2190+1	.1845-1	.778+1	.00	151	.0
116	210	3	07	16	3	27	21.15			-.4658+1			-.3827+1	-.2540+1	-.2150-2	.805+1	.01	132	.0
116	216	3	07	16	3	24	1.77		•	-.11[illegible]+4*			-.3291+4*	.2394+3*	.6860+1*	.370+4	7.96	148	.3
116	215	3	07	16	3	18	22.33		•	-.2602+4*			-.2551+4*	.1840+4*	.1042+1*	.484+4	7.97	129	.3
116	216	3	07	16	1	59	59.63			-.8700+1			.0088-1	.2557+1	.1347-1	.281+1	-.00	158	-.0
116	215	3	07	16		2	5.37			-.8682+1			-.1563+1	.1500+1	.1247-1	.124+2	-.01	140	-.0
116	216	3	07	15	23	59	5.72			-.5660+1			-.5461+1	-.3040+1	.4377-2	.101+2	-.01	130	-.0
116	216	3	07	15	23	52	36.08			-.2523+1			-.6100+1	-.3422+1	-.2244-1	.792+1	-.01	107	-.0
116	210	3	07	15	23	19	15.67			-.6233+1			-.7084+1	-.1295+1	-.1740-2	.113+2	-.01	112	-.0
116	210	3	07	15	22	7	23.27			-.5561+1			-.7345+1	-.5817+1	-.1147-1	.122+2	-.01	102	-.0
116	215	3	07	15	3	22	52.11		•	.1325+2			-.9930+1	-.8300+1	-.197348*	.227+2	.04	164	-.0
116	210	3	07	15	4	14	3.21			-.4716+1			-.1764+2	.1632+1	.6910-1	.189+2	.04	140	.0
116	216	3	07	15	1	37	43.47			.8627+1			-.4254+1	-.1171+2	-.6704-1	.173+2	.02	159	.0
116	216	3	07	15	1	32	24.93			.3697+1			.8216+1	-.8832+1	.2751-2	.117+2	.02	141	-.0
116	216	3	07	14	23	41	14.53			-.6162+1			.6984+1	.2196+1	.1228-1	.112+2	.01	115	-.0
116	216	3	07	14	23	38	54.70			-.8822+1			.9053+1	-.1216+1	.6392-2	.154+2	.02	107	-.0
116	210	3	07	14	23	37	24.88			-.1004+2			.4057+1	-.267120	.1548-2	.147+2	.01	104	-.0
116	215	3	07	14	2	1	34.90			-.4424+1			.2863+1	.272413	.5868-2	.688+1	.00	73	-.0
116	216	3	07	14	15	13	30.46			-.6606+1			.2686+1	-.3491+1	.8932-2	.101+2	.00	59	-.0
116	215	3	07	14	15	7	27.51		•	-.4682+1			-.2734+1	.5271+2*	-.104913*	.531+2	.05	38	-.1
116	210	3	07	14	15	5	47.69			-.1118+2			.1319+1	.4942+1	-.2113-1	.166+2	.02	32	-.0
116	216	3	07	14	14	37	58.92			-.5995+1			-.2654+1	.2444+1	-.6345-2	.914+1	.00	32	.0
116	210	3	07	14	14	36	29.[illegible]7			-.4620+1			-.360414	.3731+1	-.1172-1	.754+1	.00	26	.0
116	215	3	07	14	3	3	23.61			-.7083+1			-.4511+1	-.226646	.4363-2	.121+2	.00	143	.0
116	210	3	07	14	2	59	31.16			-.944 +1			-.5635+1	-.1866-1	.6764-2	.147+2	.02	130	.0
116	216	3	07	14	1	16	12.28			-.7568+1			.5093-1	-.934041	.5831-2	.107+2	-.00	132	-.0
116	215	3	07	14	1	12	25.52			-.7790+1			-.4548+1	.2257+1	-.4555-3	.121+2	.00	119	-.0
116	215	3	07	13	23	27	27.30			-.844 +1			-.8062+1	.440387	-.1204-1	.144+2	-.01	115	-.0
116	216	3	07	13	23	25	33.53			-.1094+2			-.9917+1	-.2062+1	-.1333-1	.185+2	-.01	108	-.0
116	210	3	07	13	17	59	33.23			-.9749+1			-.9089+1	.1988+1	-.1617-1	.167+2	-.02	58	-.0
116	215	3	07	13	17	55	55.73			-.6087+1			-.5919+1	-.2394+1	-.8843-2	.117+2	-.00	45	-.0
116	215	3	07	13	16	17	3.39		•	.8476+1			-.9641+1	-.1921+3*	.7567-2	.192+3	.21	63	-.2
116	216	3	07	13	15	12	53.78			.915047			-.1454+2	-.8352+1	.4311-1	.168+2	.03	46	.0
116	216	3	07	13	4	45	4.0[illegible]			-.3012+1			-.9724+1	-.870251	.1753-1	.100+2	.01	149	.0
116	215	3	07	13	4	34	34.65			-.2767+1			-.9412+1	-.3281+1	.1970-1	.107+2	.02	147	.0
116	210	3	07	13	[illegible]	33	34.77			-.3057+1			-.7381+1	-.3123+1	.1437-1	.977+1	.01	143	.0
116	215	3	07	13	2	49	58.95			-.2414+1			-.2594+1	-.5993+1	.3014-1	.737+1	.00	144	.0
116	210	3	07	13	2	42	49.72			-.6924+1			-.2422+1	-.3192+1	-.2212-1	.104+2	.01	119	-.0
116	216	3	07	13	1	5	59.24			-.156 +1			.1472+1	-.1009+1	.1355-1	.283+1	.00	142	-.0
116	216	3	07	13		50	10.99			-.6423+1			-.201077	-.3234+1	-.4107-2	.964+1	.00	120	-.0
116	216	3	07	12	23	10	42.51			-.2731+1			-.7509+1	.1109+1	.2182-2	.851+1	-.01	104	-.0
116	210	3	07	12	23	11	39.29		•	.116[illegible]+2			-.6458+2*	-.1810+2	-.3960-1	.690+2	-.14	107	.1
116	328	3	07	17	13	45	7.59	1		.196 +2			.1178+1	.5155+1	.2046-1	.282+2	-.01	66	-.1
116	328	3	07	17	13	20	26.98	1		.1092+1			-.9551+1	-.4086+1	-.1109-1	.105+2	.02	85	.0
116	328	3	07	17	15	35	57.41	2	*	-.4445+2*			-.3542+1	-.5911+1	-.120014*	.628+2	-.04	90	.2

FIGURE 7. SAMPLE OUTPUT DATA - SATELLITE 116, CASE 3 (3 OF 5)

SIMPLIFIED GENERAL PERTURBATION DIFFERENTIAL CORRECTION DECEMBER 19, 1963 PAGE 27

SATELLITE NO. SATELLITE NAME: ELEMENT SET NO. TIME OF EPOCH 193.9449674

TAG NO.	STA NO.	Y	MM	DD	TIME HH	MM	SS.SS	R A	R J	RANGE RES. KM.	RT ASCEN RES. KM.	DECL. RES. KM.	AZIMUTH RES. KM.	ELEV. RES. KM.	RR. RES. KM./SEC.	VECTOR MAG. KM.	DELTA T MIN.	U DEG.	BETA DEG.
116	329	3	07	16	23	50	2.97	1	*	-.1397+2			.3179+1	.3935+1	.4438+1*	.203+2	.03	51	.0
116	329	3	07	17	1	38	2.73	1	*	-.341 +1			.6596+1	.1357+2	.4788+1*	.158+2	.02	65	.0
116	329	3	07	17	3	24	9.87	1	*	-.3010+2*			.41[illegible]+1	.3705+2*	.4446+1*	.566+2	.09	74	.0
116	329	3	07	17	4	24	25.67	1	*	.3129+2*			.3796+1	-.1824+2	.4451+1*	.480+2	-.08	74	.0
116	329	3	07	17	3	25	16.21	1	*	.1011+2			-.4828+1	-.1155+2	.4432+1*	.190+2	-.03	77	-.0
116	329	3	07	17	5	11	10.28	1	*	-.1448+2			-.5390+0	-.3409+1	.4371+1*	.207+2	.02	84	.0
116	329	3	07	17	5	12	3.40	1	*	-.2106+2			-.72[illegible]3+0	.5770+1	.4316+1*	.303+2	.04	88	.0
116	329	3	07	17	6	59	16.27	1	*	.913 +1			.1363+2	-.2415+1	.3218+1*	.169+2	-.03	99	.0

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SATELLITE NO. SATELLITE NAME: ELEMENT SET NO. TIME OF EPOCH 193.9449674

CASE NO.	RMS KM.	RMSR KM/SEC	DELTA N/N	DELTA AYN	DELTA AYN	DELTA UO	DELTA NODE	DELTA I	DELTA CO
4	.8833824+1	.2159710-1	.2703652?-6	-.45442-4	-.70124-5	.10559-3	.50071-4	.27566-4	

CORRECTED ELEMENTS

REV. NO.	CASE NO.	L DEGREES	TO DAYS	A EARTH RADII	E	I DEG.	NODE DEG.	OMEGA DEG.	CO DA/REV**2	PER ALT ST. MI.	PA MINUTES
	5	93.97215	193.94496	1.1473394	.00746	66.892	93.429	142.048	.0000000	549.9	103.818

FIGURE 7. SAMPLE OUTPUT DATA - SATELLITE 116, CASE 3 (4 OF 5)

SIMPLIFIED GENERAL PERTURBATION DIFFERENTIAL CORRECTION DECEMBER 19, 1963

SATELLITE NO. SATELLITE NAME: ELEMENT SET NO. 0 TIME OF EPOCH 103.9449463

OLD RMS KM.	NEW RMS KM.	OLD RMS2 KM/SEC	NEW RMS2 KM/SEC	DELTA N/N	DELTA AXN	DELTA AYN	DELTA MO	DELTA NODE	DELTA I	DELTA CO
.410+3	.883+1	.3.259	.215-1	.2036597-6	-.45642-4	-.7.424-5	.10659-3	.50071-4	.27566-4	

NO. OF RESIDUALS USED = 354 NO. OF RESIDUALS REJECTED = 56

	REV. NO.	SET NO.	L DEGREES	TO DAYS	A EARTH RADII	E	I DEG.	NODE DEG.	OMEGA DEG.	CO DA/REV**2	PER ALT ST. MI.	PA MINUTES
OLD ELEMENTS		0	88.45246	103.94460	1.1500850	.01114	67.272	87.772	123.982		544.0	104.190
NEW ELEMENTS			93.97215	103.94496	1.1473394	.00746	66.802	93.429	142.088	.0000000	549.9	103.818
NEW ELEMENTS		1	93.97215	103.94496	1.1473394	.00746	66.802	93.429	142.088	.0000000	549.9	103.818

NODE DOT = -2.425 DEG/DAY, OMEGA DOT = -.590 DEG/DAY

ALL ELEMENTS CORRECTED TO FIRST ATTEMPT

FLEMENTS NOT REPLACED, MANUAL RUN

END ICLS3
12-19-11-16.2

FIGURE 7. SAMPLE OUTPUT DATA - SATELLITE 116, CASE 3 (5 OF 5)

REFERENCES

1. Hilton, C. G., "The SPADATS Mathematical Model," Aeronutronic Publication No. U-2202, 5 August 1963.

2. "Space Detection and Tracking System (SPADATS) Programming Document" (Revised), Aeronutronic Publication No. U-1691, 1 October 1962.

3. "Spacetrack Operators Manual," System Development Corporation Publication TM-LX-37, 14 February 1963.